第５章「非 定 常 な Ａ Ｒ Ｍ Ａ モ デ ル」

１．ARIMA(p,d,q) モデル

$$\phi(L)(1-L)^d y_t = \phi(L)\triangle^d y_t = m + \theta(L)\varepsilon_t \quad (t = 1, 2, \cdots, T)$$

Note. 初期値 $y_j = 0$ $(j \leq 0)$．$1 - L = \triangle$ は，１期前との階差を取る階差オペレータ.

$$\triangle y_t = y_t - y_{t-1}, \quad \triangle^2 y_t = \triangle(y_t - y_{t-1}) = y_t - 2y_{t-1} + y_{t-2}, \text{ etc.}$$

２．ARIMA(p,d,q) モデルの h 期先予測

$\hat{y}_t(h)$:　時点 t までの観測値が与えられたときの y_{t+h} の BLUP

例１： ARIMA(1,1,0) $(1-\phi_1 L)(1-L)y_t = m + \varepsilon_t$ の場合

$x_t = (1-L)y_t \sim \text{AR}(1)$ より，

$$\begin{aligned} x_{t+h} &= m + \phi_1 x_{t+h-1} + \varepsilon_{t+h} \\ y_{t+h} &= y_{t+h-1} + x_{t+h} \\ \hat{x}_t(h) &= m + \phi_1 \hat{x}_t(h-1) \end{aligned}$$

であるから，

$$\begin{aligned} \hat{y}_t(h) &= \hat{y}_t(h-1) + \hat{x}_t(h) \\ &= \hat{y}_t(h-2) + \hat{x}_t(h-1) + \hat{x}_t(h) \\ &= y_t + \hat{x}_t(1) + \hat{x}_t(2) + \cdots + \hat{x}_t(h) \end{aligned}$$

特に，$\phi_1 = 0$ のとき（$\Leftrightarrow y_t = y_{t-1} + m + \varepsilon_t$: ドリフトのあるランダム・ウォーク）は，

$$\hat{y}_t(h) = y_t + hm$$

例２： ARIMA(0,1,1) $(1-L)y_t = m + \varepsilon_t - \theta_1 \varepsilon_{t-1}$ の場合

$y_{t+h} = y_{t+h-1} + m + \varepsilon_{t+h} - \theta_1 \varepsilon_{t+h-1}$ より，

$$\begin{aligned} \hat{y}_t(h) &= \hat{y}_t(h-1) + m \\ &= \hat{y}_t(h-2) + 2m \\ &= \hat{y}_t(1) + (h-1)m \\ &= y_t + m - \theta_1 \varepsilon_t + (h-1)m \\ &= y_t - \theta_1 \varepsilon_t + hm \end{aligned}$$

３．ARIMA(p,d,q) モデルの推定

階差の次数 d を与えて，$x_t = (1-L)^d y_t$ を定常な ARMA(p,q) モデルと想定することにより，以前の場合のように，次の手続きに従う．

(1) Identification (同定，特定化)
⇒ (2) Estimation (推定)
⇒ (3) Diagnostic Checking (診断)

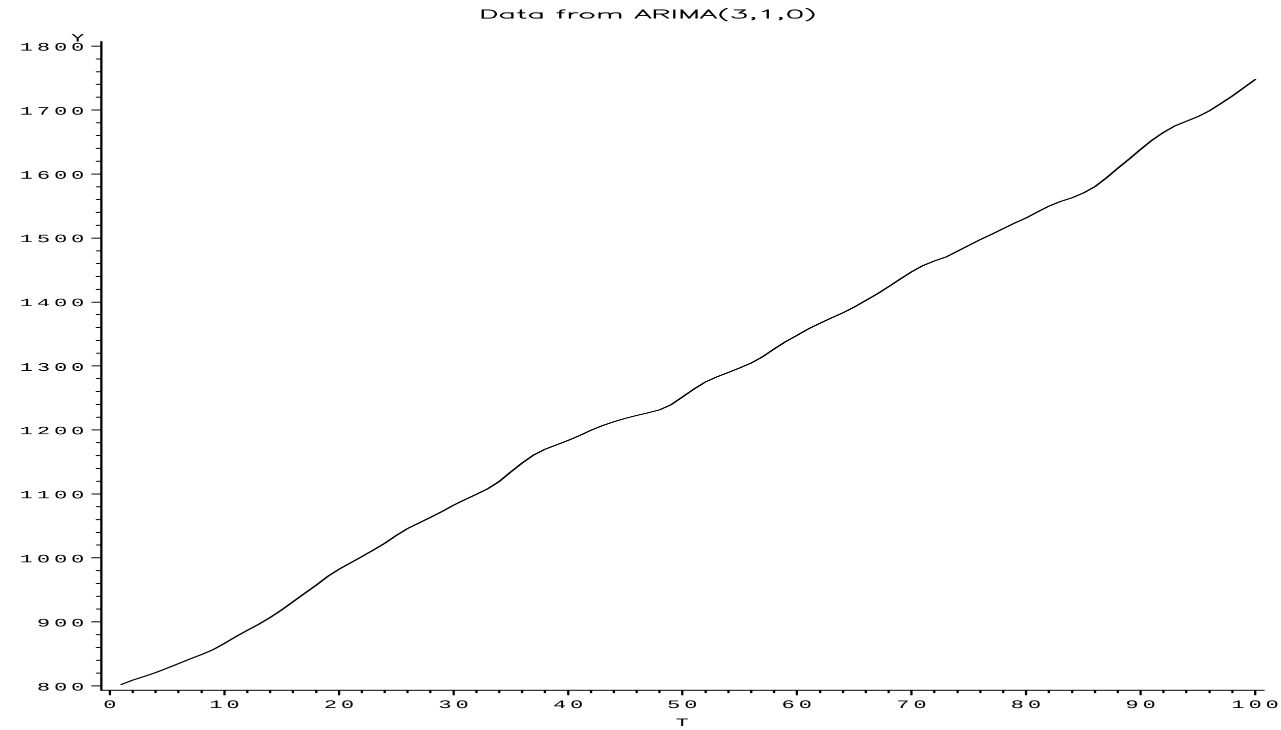
例３： ARIMA(3,1,0) の場合

DGP:　$(1-L)y_t = m + \phi_1 y_{t-1} + \phi_2 y_{t-2} + \phi_3 y_{t-3} + \varepsilon_t$

$m = 1.5,\quad \phi_1 = 1.55,\quad \phi_2 = -1.1,\quad \phi_3 = 0.4,\quad \sigma^2 = 1$

この DGP からの 100 個のデータは次の通りである．

802.177	809.163	814.839	820.802	827.643	834.788	842.342	849.178
856.621	866.307	876.832	886.878	896.390	906.939	918.392	931.342
944.359	957.392	971.020	982.538	992.344	1002.095	1012.419	1022.823
1034.802	1045.759	1054.452	1063.070	1072.674	1082.538	1091.350	1099.558
1108.074	1119.700	1134.130	1148.299	1160.658	1169.858	1176.575	1183.751
1191.479	1199.574	1206.670	1212.833	1217.987	1222.633	1226.798	1231.175
1239.375	1251.209	1264.276	1275.071	1283.249	1289.773	1296.970	1304.470
1314.453	1326.712	1337.721	1347.805	1357.854	1367.013	1375.197	1383.239
1392.210	1402.433	1412.681	1423.867	1435.772	1446.937	1456.857	1463.893
1470.615	1479.458	1488.546	1497.603	1506.069	1514.708	1523.282	1531.275
1540.805	1549.829	1556.947	1563.026	1570.442	1580.490	1594.034	1608.999
1623.594	1638.496	1652.801	1665.182	1675.188	1682.486	1689.753	1699.085
1710.434	1722.020	1734.755	1747.873				




```
                     Data from ARIMA(3,1,0)                     1
                Name of variable = Y.
                Period(s) of Differencing = 1.
                Mean of working series = 9.552485
                Standard deviation     = 2.494418
                Number of observations =       99
                NOTE: The first observation was eliminated by
                      differencing.

                        Autocorrelations

Lag Covar   Corr     -1 9 8 7 6 5 4 3 2 1 0 1 2 3 4 5 6 7 8 9 1
  0 6.22212 1.00000  |                    |********************|
  1 5.05534 0.81248  |                .   |****************    |
  2 2.80772 0.45125  |              .     |*********           |
  3 0.85294 0.13708  |             .      |***   .             |
  4  -0.267 -.04291  |             .     *|      .             |
  5 -0.4917 -.07903  |             .    **|      .             |
  6 -0.2209 -.03550  |             .     *|      .             |
  7 0.11116 0.01787  |             .      |      .             |
  8 0.30268 0.04865  |             .      |*     .             |
  9 0.28521 0.04584  |             .      |*     .             |
 10 -0.0542 -.00871  |             .      |      .             |
 11 -0.5501 -.08841  |             .    **|      .             |
 12 -1.0392 -.16702  |             .   ***|      .             |
 13 -1.3603 -.21862  |             .  ****|      .             |
 14 -1.2794 -.20561  |             .  ****|      .             |
 15 -0.7926 -.12739  |             .   ***|      .             |
```

```
16 -0.2985 -.04797  |              .     *|      .            |
17 -0.0283 -.00454  |              .      |      .            |
18 0.06026 0.00969  |              .      |      .            |
19 -0.0341 -.00547  |              .      |      .            |
20 -0.1988 -.03195  |              .     *|      .            |
21 -0.3276 -.05266  |              .     *|      .            |
22 -0.4841 -.07780  |              .    **|      .            |
23 -0.4845 -.07787  |              .    **|      .            |
24 -0.4374 -.07030  |              .     *|      .            |
                              "." marks two standard errors

             Autocorrelation Check for White Noise

 To   Chi                    Autocorrelations
Lag  Square DF   Prob
  6   91.29  6  0.000  0.812  0.451  0.137 -0.043 -0.079 -0.036
 12   95.92 12  0.000  0.018  0.049  0.046 -0.009 -0.088 -0.167
 18  108.68 18  0.000 -0.219 -0.206 -0.127 -0.048 -0.005  0.010
 24  111.41 24  0.000 -0.005 -0.032 -0.053 -0.078 -0.078 -0.070

                 Maximum Likelihood Estimation

                                Approx.
       Parameter   Estimate    Std Error   T Ratio  Lag
       MU           9.59343      0.48834     19.65   0
       AR1,1        1.53276      0.09777     15.68   1
       AR1,2       -1.01510      0.15253     -6.66   2
       AR1,3        0.27086      0.09847      2.75   3

       Constant Estimate  =  2.0288504

       Variance  Estimate = 1.11043167
       Std Error Estimate = 1.05377022
       AIC                = 297.688043
       SBC                = 308.068522
       Number of Residuals=         99

                 Correlations of the Estimates


   Parameter          MU       AR1,1       AR1,2       AR1,3

   MU              1.000       0.019      -0.010       0.031
   AR1,1           0.019       1.000      -0.873       0.650
   AR1,2          -0.010      -0.873       1.000      -0.871
   AR1,3           0.031       0.650      -0.871       1.000
```

Autocorrelation Check of Residuals

To Lag	Chi Square	DF	Prob	Autocorrelations					
6	2.02	3	0.567	0.026	-0.067	0.091	-0.074	0.015	0.015
12	4.47	9	0.878	-0.047	-0.037	0.124	-0.053	-0.012	0.012
18	8.64	15	0.896	-0.084	-0.125	0.017	0.074	-0.079	0.021
24	14.86	21	0.830	-0.004	-0.041	0.119	-0.157	-0.035	0.078

Model for variable Y

Estimated Mean = 9.59342697
Period(s) of Differencing = 1.

Autoregressive Factors
Factor 1: 1 - 1.5328 B**(1) + 1.0151 B**(2)
- 0.27086 B**(3)

Forecasts for variable Y

Obs	Forecast	Std Error	Lower 95%	Upper 95%
101	1760.2194	1.0538	1758.1541	1762.2848
102	1771.3057	2.8694	1765.6817	1776.9297
103	1781.3473	4.9838	1771.5792	1791.1155
104	1790.8581	6.9722	1777.1927	1804.5235
105	1800.2742	8.6465	1783.3274	1817.2210
106	1809.8011	10.0112	1790.1795	1829.4227
107	1819.4502	11.1539	1797.5889	1841.3115
108	1829.1485	12.1646	1805.3063	1852.9907
109	1838.8281	13.1030	1813.1467	1864.5094
110	1848.4622	13.9967	1821.0292	1875.8952
111	1858.0589	14.8526	1828.9484	1887.1695
112	1867.6395	15.6695	1836.9278	1898.3512
113	1877.2208	16.4468	1844.9857	1909.4560
114	1886.8097	17.1869	1853.1241	1920.4953
115	1896.4049	17.8942	1861.3329	1931.4770
116	1906.0025	18.5738	1869.5985	1942.4064
117	1915.5991	19.2294	1877.9101	1953.2882
118	1925.1938	19.8640	1886.2610	1964.1266
119	1934.7871	20.4794	1894.6481	1974.9260
120	1944.3797	21.0771	1903.0694	1985.6900

(参考１）データ生成のための SAS プログラム

```
filename out 'chap5-1.dat';    データの出力先を確保
data ar3;y=0;                  初期値を設定
do i=1 to 200;                 全部で 200 個のデータ
e=rannor(2784322);             正規乱数の発生
lags=sum(1.55*lag1y,-1.1*lag2y,0.4*lag3y,e);
y=1.5+lags;
lag3y=lag2y;lag2y=lag1y;lag1y=y;
if i>100 then output;          後半の 100 個のみを取り出す
end;
data b;set ar3;
file out;put y f10.3;          小数点以下３桁（全体 10 桁）でデータを
                               cha5-1.dat に書き出す
```

(参考２）ARMA モデル分析のための SAS プログラム

```
options ls=65 ps=50;       出力サイズの設定（ls=列文字数，ps=行文字数）
filename in1 'chap5-1.dat';     データの読み込み先を指定
filename graph1 'chap5-1.ps';   グラフの出力先を確保
goptions device=psepsf gsfmode=replace gsfname=graph1;
data ar3;infile in1;input y;
t=_n_;
symbol i=join l=1 v=none;
proc gplot;plot y*t;
title f=simplex h=1.5 'Data from ARIMA(3,1,0)';
proc arima;
  identify var=y(1);           y(1) は，y の１回階差を分析
  estimate p=3 method=ml;      AR(3) を最尤法（method=ml）で推定
  forecast out=b1 lead=20;     20 期先までの予測
  estimate p=2 method=ml;      AR(2) を最尤法で推定
  forecast out=b2 lead=20;     20 期先まで予測
  estimate p=2 q=1 method=ml;  AR(2,1) を最尤法で予測
  forecast out=b3 lead=20;     20 期先まで予測
```